A Novel Heuristic Filter Based on Ant Colony Optimization for Non-linear Systems State Estimation

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Abstract. A new heuristic filter, called Continuous Ant Colony Filter, is proposed for non-linear systems state estimation. The new filter formulates the states estimation problem as a stochastic dynamic optimization problem and utilizes a colony of ants to find and track the best estimation. The ants search the state space dynamically in a similar scheme to the optimization algorithm, known as Continuous Ant Colony System. The performance of the new filter is evaluated for a nonlinear benchmark and the results are compared with those of Extended Kalman Filter and Particle Filter, showing improvements in terms of estimation accuracy.

Keywords: Non-linear Systems State Estimation, Heuristic Filter, Ant Colony Optimization, Particle Filter.

1 Introduction

In many engineering applications, one needs to estimate the states of a dynamic system. A state estimation problem is defined as follows: given the mathematical model of a dynamic system, it is desired to estimate the time-varying states using a noisy measurement. Estimation problems are often categorized as prediction, filtering and smoothing, depending on intended objectives and the available observations[1]. Here, the domain of focus is filtering, which is usually referred as the extraction of true signal from the observations. Filters are usually minimizing a given objective function, while they are working. Such filters are called optimal filters[2].

Optimal filters are categorized to recursive and batch filters[1][3]. A batch filter, e.g. least square filter, uses the complete history of measurements to estimate unknown states. A Recursive filters, in comparison, has the ability to receive and process measurements sequentially. Recursive filters consist of two essentially stages: prediction and update[3]. Prediction uses the estimated states of the previous time step to produce an initial estimate of the current step. This stage is also known as the priori state estimation because it does not use the observations, obtained in the current time step. In update stage, the priori state

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estimation is combined with the current observation to refine the state estimation. This improved estimation is also termed as the posterior state estimation. The dynamic states can be estimated using the posterior Probability Density Function (PDF), obtained based on the received measurement. If either the system or measurement model is nonlinear, the posterior PDF will not be Gaussian, even if the measurement and the process noises are assumed to be Gaussian.

Several recursive filters can be found within the literature, the most well-known of which are Kalman Filter (KF)[4], Extended Kalman Filter (EKF)[5], Unscented Kalman Filter (UKF)[6], Particle Filter (PF) [7] and etc.

Recursive filters can also be categorized to linear and nonlinear filters[1][3]. In a linear filter, such as KF, both system and measurement models are linear. KF assumes the posterior PDF to be Gaussian and can be characterized by a mean and a covariance. In opposite, a nonlinear filter, such as EKF, UKF and PF, is used to estimate the states of a nonlinear dynamic system when either the system or the measurement model is nonlinear.

Analytical approximation and states sampling are two common approaches in nonlinear filtering. In the first approach, the nonlinear functions of the mathematical model are linearized and then a linear filter such as KF is used as well. EKF is an example of filters, work based on analytical approximation. Unlike to EKF, UKF is a sample based filter. It does not approximate the nonlinear mathematical model. Instead, it approximates the posterior PDF by a set of deterministically chosen samples. UKF is also referred to as a linear regression Kalman filter, because it is based on statistical linearization rather than analytical ones[3].

The authors categorize sample based filters to mathematical and heuristic approaches. UKF can be taken a mathematical sample based filter to account, since it uses a deterministic sampling process, the general estimation mathematics and the mathematical operators such as unscented transform. In comparison, there are several sample based filters that utilize heuristic algorithms to sample the particles and to improve the position of them. These filters can be called heuristic filters.

PF is an example of heuristic filters. It works based on point mass (or particle) representation of the probability densities[8]. Unlike to UKF, PF represents the required posterior PDF by a set of random samples instead of deterministic ones. Also, it uses a re-sampling procedure to reduce the degeneracy of particle set. The standard re-sampling procedure copies the important particles and discards insignificant ones based on their fitness. This strategy suffers from the gradual loss of diversity among the particles, known as sample impoverishment. Different re-sampling strategies have been proposed in the literature, such as Binary Search[9], Systematic Re-sampling[10] and Residual Re-sampling[8].

PF has several variants with different sampling and re-sampling procedures. All sampling procedures, utilized in PFs, can be derived from the Sequential Importance Sampling (SIS) algorithm[11] by the appropriate choice of importance sampling density[3]. The combination of SIS and Systematic Re-sampling is called Generic PF (GPF)[3]. Sampling Importance Resampling (SIR) filter[9], Boostrap Particle Filter (BPF)[12], Auxiliary Sampling Importance Resampling (ASIR) filter[13], Unscented Particle Filters (UPF)[14], Extended Particle Filters (EPF) [10], Multiple-model Particle Filter (MMPF)[12], Regularized Particle Filter (RPF)[15] and Markov Chain Monte Carlo (MCMC)[16] are example variants of PF.

Recently, some heuristic optimization algorithms have been augmented with PFs. Genetic Algoritm (GA) and PF have been combined to increase diversity of samples after re-sampling[17][18]. Simulated Annealing (SA) has been introduced into PF to improve its performance[19]. A local search method has been inserted into particle filter to reduce the sample size and improve the efficiency[20]. Particle Swarm Optimization (PSO) has been introduced into PF to solve the particle impoverishment and sample size dependency problems[21]. Ant Colony Optimization (ACO) has been utilized to improve the re-sampling process[22][23]. ACO for Real domains (ACOR) has been incorporated into PF to optimaize the sampling process[24].

The state estimation problem can be formulated as a stochastic dynamic optimization problem. Therefore, different ideas of heuristic optimization can be extended and modified to solve this problem. Here, the authors have proposed a new heuristic filter for non-linear systems state estimation, based on their previously developed metaheuristic, known as Continuous Ant Colony System (CACS)[25]. The proposed filter is called Continuous Ant Colony Filter (CACF). CACF can be categorized as a heuristic sample based filter. It utilizes a colony of moving ants, the average positions of which is returned as the current estimation. In this filter the estimation of the current states is formulated as a stochastic dynamic optimization problem and an optimization algorithm, based on CACS is utilized to iteratively find and track the best estimation.

This paper is organized as follows: A state estimation problem is formulated in section 2. Section 3 is devoted to a detailed description of the new estimation algorithm. The experimental results are provided in section 4. The final Conclusion is made in section 5.

2 Problem Formulation

The problem is to estimate the states of a nonlinear dynamic system. Discrete-time state space approach is utilized to model the evolution of system and the noisy measurements. The states are assumed to be evolved according to the following stochastic model:

$$\boldsymbol{x}_k = \boldsymbol{f}_k(\boldsymbol{x}_{k-1}, \boldsymbol{v}_{k-1}) \tag{1}$$

where $\boldsymbol{f}_k$ is a known, possibly nonlinear function of the state vector $\boldsymbol{x}_{k-1}$, $\boldsymbol{v}_{k-1}$ is referred to as the process noise, and k is the time counter. The objective of a nonlinear filter is to recursively estimate $\boldsymbol{x}_k$ from the available measurements, $\boldsymbol{z}_k$. The measurements are related to the states via the measurement equation, stated as follows:

$$\boldsymbol{z}_k = \boldsymbol{h}_k(\boldsymbol{x}_k, \boldsymbol{w}_k) \tag{2}$$

where $\boldsymbol{h}_k$ is a known, possibly nonlinear function and $\boldsymbol{w}_k$ is the measurement noise. The noise sequences, $\boldsymbol{v}_k$ and $\boldsymbol{w}_k$, are assumed to be white, with known probability density functions and mutually independent. A graphical illustration of the evolution and the measurement models can be depicted as in Fig. 1. The initial state, x_0, is assumed to have a known PDF $p(x_0)$ and to be independent of the noise sequence.

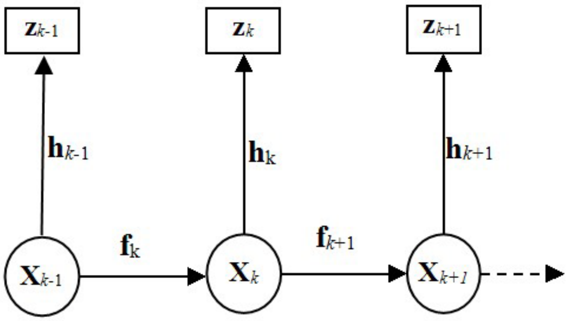

Fig. 1. Process and measurement models of a dynamic system

3 Continuous Ant Colony Filter

In this section, the new heuristic filter is introduced as a tool for nonlinear systems state estimation. First, the general structure of the filter is presented. Then each constructive module is discussed in detail.

3.1 General Setting Out of the Algorithm

Fig. 2 shows the general iterative structure of CACF. A high level description of the sequential steps is shown in this figure. The parameters of CACF and the initial position of ants are set during the initialization, as discussed in the section 3.2.

CACF has two loops: a main outer loop, iterating every time a new measurement is entered, and an inner loop, iterates to find the best estimation of the current states, corresponding to the entered measurement. The inner loop propagates the initial distribution of ants, at first. Then, the output, estimated by each ant, is made. The estimated outputs are compared with the real measurement and each ant is assigned a cost, based on the quality of its estimation. Ants use their experience to update the state space pheromone distribution. As in CACS[25], a Gaussian function is utilized to model the pheromone distribution over the continuous state space. Ants use this pheromone distribution to move from their current position toward the minimum cost destinations. The destinations are chosen using a normal PDF. The inner loop is terminated after a predefined number of iterations. Finally, the current state estimation is made using a mean operator. In the following subsections, these steps are discussed in detail.

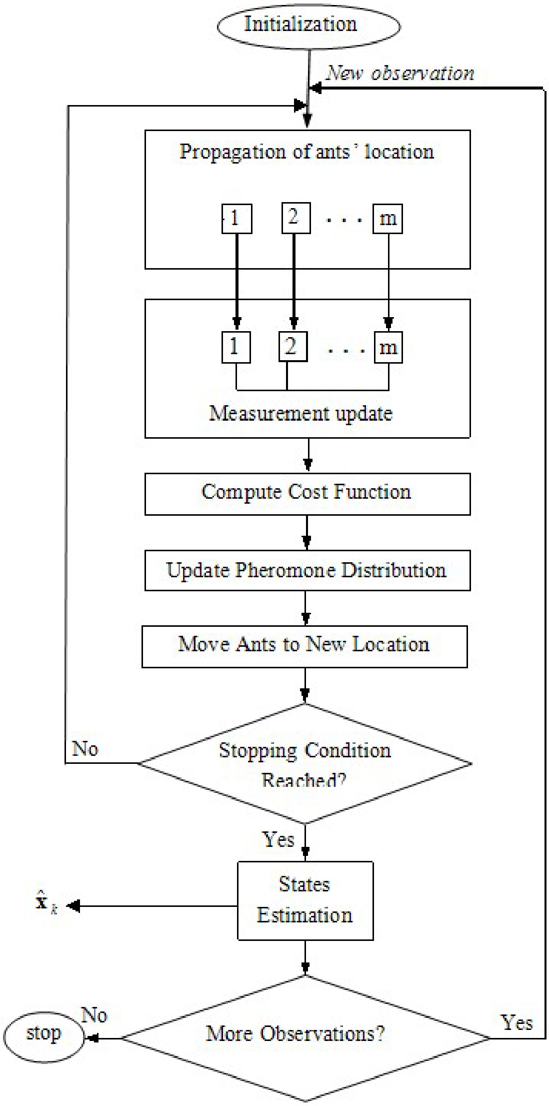

Fig. 2. Continuous ant colony filter (CACF) algorithm

3.2 Initialization

The new algorithm has some control parameters that must be set before the execution of the algorithm. The inner loop is terminated after q iterations. Moreover, the initial position of ants is initialized using a uniform random generator.

3.3 Propagation and Measurement Update

At the beginning of the i-th iteration of the inner loop, the position of ant j at time $k-1$, defined as, $\boldsymbol{x}_k^{i,j}$ is propagated as follows:

$$\hat{\boldsymbol{x}}_k^{i,j} = \boldsymbol{f}_k(\boldsymbol{x}_{k-1}^{i,j}, \boldsymbol{v}_{k-1}^{i,j}) \tag{3}$$

Then the current output, estimated in iteration i by ant j at time k, noted by $\hat{\boldsymbol{z}}_k^{i,j}$, is calculated as follows:

$$\hat{\boldsymbol{z}}_k^{i,j} = \boldsymbol{h}_k(\hat{\boldsymbol{x}}_k^{i,j}) \tag{4}$$

3.4 Compute Cost Function

Each ant is assigned a cost, based on the quality of its current position. The cost function is defined as the square error between the estimated output, $\hat{\boldsymbol{z}}_k^{i,j}$, and the real measurement, $\boldsymbol{z}_k$. Therefore, the cost, assigned in iteration i to ant j at time k, is calculated as follows:

$$f_k^{i,j} = (\hat{\boldsymbol{z}}_k^{i,j} - \boldsymbol{z}_k)^2 \tag{5}$$

In this way, the cost function is calculated in different points of the state space and some knowledge about the problem is acquired, used later to update the pheromone distribution.

3.5 Update Pheromone Distribution

CACF utilizes the same pheromone model and pheromone updating rule, as in CACS[25]. During any iteration, phermone distribution will be updated using the acquired knowledge from the evaluated points by ants. Pheromone updating rule of CACF can be stated as follows: during any iteration, the cost is calculated for the new points, explored by the ants. Then, the best point, found up to the ith iteration, at time $k-1$, is assigned to $\boldsymbol{x}_{k-1,min}^i$.

Also, the standard deviation of the pheromone distribution $(\boldsymbol{\sigma}_{k-1}^i)$ is updated based on the cost of the evaluated points and the aggregation of those points around $\boldsymbol{x}_{k-1,min}^i$. To satisfy simultaneously the fitness and aggregation criteria, the concept of weighted variance, proposed in [25], is defined as follows:

$$(\boldsymbol{\sigma}_{k-1}^i)^2 = \frac{\sum_{j=1}^{m} \frac{1}{f_{k-1}^{i,j} - f_{k-1,min}^i} (\boldsymbol{x}_{k-1}^{i,j} - \boldsymbol{x}_{k-1,min}^i)^2}{\sum_{i=1}^{m} \frac{1}{f_{k-1}^{i,j} - f_{k-1,min}^i}} \tag{6}$$

Here, m is the number of ants. This strategy means that the center of region, discovered during the subsequent iteration, is the last best point and the narrowness of its width depends on the aggregation of the other competitors around the best point. It should be noted that after termination of the inner loop, the standard deviation of the pheromone distribution is increased by an Expansion Factor (EF) to increase the exploration of the filter when the new measurement is entered.

3.6 Movement of the Ants

During any iteration, ants move from their current position to their destination using the current pheromone distribution. Pheromone distribution is modeled using a normal PDF, the center of which is the best point ($\boldsymbol{x}^i_{k-1,min}$), found from the first iteration and its variance depends on the aggregation of other ants around the best one. Normal PDF permits all points of the continuous state space to be chosen, either close to or far from the best point. As stated in section 3.2, in the first iteration, the position of ants is initialized using a uniform random generator, whereas for all subsequent iterations, ants chose their destination using the updated pheromone distribution, based on equation (6).

3.7 Stopping Condition

CACF has two loops, each with its own specific stopping condition. The inner loop stops when the maximum number of iterations (q) is reached. The outer loop stops when the measurements are finished.

3.8 States Estimation

After termination of the inner loop, the states are estimated based on the average position of top ants:

$$\hat{\boldsymbol{x}}_k = \frac{1}{m_t} \sum_{j=1}^{m_t} \hat{\boldsymbol{x}}_k^{q,j} \tag{7}$$

where m_t denote number of top ants.

4 Results and Discussion

In this section, the performance of the new filter is investigated for a benchmark, taken from the literature. This study is intended to provide a comparison of the proposed state estimation method with more established approaches. Table 1 shows the tuned parameters of CACF.

A nonlinear single variable economic model[14], defined by (8) and (9), is employed to test the performance of CACF and compare it with that of EKF and Generic PF.

$$\boldsymbol{x}(t+1) = 1 + sin(4 \times 10^{-2}\pi t) + 0.5\boldsymbol{x}(t) + \boldsymbol{v}(t) \tag{8}$$

$$z(t) = \begin{cases} \frac{x(t)^2}{5} + w(t) & t \leq 30 \\ -2 + \frac{x(t)}{2} + w(t) & t > 30 \end{cases} \quad (9)$$

where $v(t)$ and $w(t)$ stand for zero-mean white noise and Gamma distribution, respectively[22]. The variance of $w(t)$ is 1×10^{-5} and the parameters k and θ of Gamma distribution are equal to 7 and 2, respectively[26].

Considering the measurement and process noises, the state sequence x_t is estimated using CACF and the results are compared with those of EKF and Generic PF, as reported in[21][22]. To make the results comparable with those of[22], the simulations are done from t = 1 to 60 and the average performance, obtained for 30 different runs are compared. Fig. 3 represents a sample output of CACF and shows that this filter can track the true signal accurately.

Table 2 shows the mean of the Root Mean Square Error (RMSE) obtained three filters. It can be observed that CACF produces comparable and even better results than EKF and Generic PF.

Table 1. Parameters of CACF

Parameter	Value
Numbers of Ants(m)	200
Maximum Number of Iterations(q)	10
Expansion Factor(EF)	2
Number of Top Ants	80

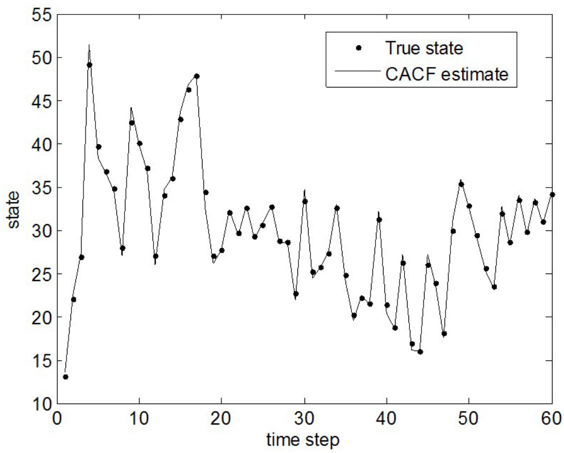

Fig. 3. The estimation history of CACF for the nonlinear single variable economic model

Table 2. Comparison of CACF with EKF and Generic PF

Filter	Mean RMS Error	RMSE Percentage (EKF=100%)
EKF[22]	0.9809	100
Generic PF[22]	0.7792	79
CACF	0.6513	66.398

5 Conclusion

In this paper a new heuristic sample based filter, was proposed for non-linear systems state estimation. The proposed filter, called CACF, models the estimation problem as a stochastic dynamic optimization problem and an optimization scheme, based on CACS, was utilized to solve this problem. CACF was tested over a benchmark to compare its results with those of EKF, as a mathematical nonlinear approach and PF, as a heuristic approach. The overall results show that CACF can properly compete with these well-known filters. One of the most important features of CACF is its simplicity.

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